1. Given a poset \( \{a, b, c, d, e\} \) with the relation defined by figure 0.1,
   
   a) Verify it is a lattice? (2 marks)
   
   b) Is \( \{a, c, d, e\} \) a sublattice of \( \{a, b, c, d, e\} \). (3 marks)

   ![Figure 0.1: Lattice](image)

2. Let \( A \) be the set of finitely generated subgroups of a group \( G \), ordered by set inclusion.
   
   a) Prove that \( \langle A, \cap \rangle \) is a meet-semilattice. (2 marks)
   
   b) Prove that \( \langle A, \subseteq, \cap \rangle \) is a lattice. (3 marks)

3. Suppose that \( \alpha \) is a well-defined proposition not containing \( \neg \). Show that the length of \( \alpha \) (the number of symbols in the string) is odd. (3 marks)

4. Prove that the binary connective \( A \downarrow B \) (neither \( A \) nor \( B \)) whose truth table is given in Figure 0.2 is adequate. (4 marks)

5. Given \( \Sigma \models \alpha \). If \( \Sigma \) is a set of valid propositions, then \( \alpha \) is also valid. (3 marks)
6. Prove or disprove the following statements. If it is false, a counterexample is needed: (10 marks)
   a) \{ ( p_1 \rightarrow ( p_2 \rightarrow p_3 ) ), p_2 \} \vdash ( p_1 \rightarrow p_3 ) \} (4 marks)
   b) \{ p \rightarrow ( q \lor r ), \neg q, \neg r \} \vdash \neg p (3 marks)
   c) \(( (P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R)))\) (3 marks)

7. Brown, Jones, and Smith are suspected of a crime. They testify as follows:
   a) Brown: Jones is guilty and Smith is innocent.
   b) Jones: If Brown is guilty then so is Smith.
   c) Smith: I’m innocent, but at least one of the others is guilty.

Represent their testimonies with propositions and show who would be the criminal. (5 marks)

8. Suppose \( \Sigma \) is a finite set of propositions. Show that every CST from \( \Sigma \) is finite. (4 marks)

9. A partial order has width at most \( n \) if every set of pairwise incomparable elements has size at most \( n \). A chain in a partial order \( < \) is simply a subset of the order that is linearly ordered by \( < \). (6 marks)
   a) Formalize partial order \( < \) with propositions. (2 marks)
   b) Prove that an infinite partial order of width with at most 4 can be divided into four chains if every finite partial suborder of width at most 4 can be so divided. (4 marks)